Colour manipulation and Colour Combination in Double-Cone Colour Space

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ABSTRACT

We derive a formula for the result of the additive mixture of two colours, in double-cone space. We use Naka-Rushton law to combine the luminance, circular weighted averaging to combine the hue and two rules of thumb to get the resulting chromatic saturation.

Keywords: Additive colour combination. Colour-space colour combination. Double-cone space. Hue, saturation, luminance.

1. INTRODUCTION

We consider ”colour-space based colour combination” (CSCC), meaning the reproduction of the colour that results when two light beams of known colours are combined, where the colours are specified as points in a given colour space. Two interesting aspects of CSCC are that, mathematically, it provides $[0, 1]^3 \subset \mathbb{R}^3$ with an operation $\oplus$ of ”addition,” that does not obey the paralellogram law since $[0, 1]^3$ is bounded. Also, it is supposed to model what happens physically when two beams of light are combined: their spectra are added in the combined beam.

In addition to other aspects of colour, the colour of a light beam is information regarding its spectral contents. You get such information through the cone receptors in the retina, and the visual system somehow ”outputs” the quale of hue saturation and luminance into your mind. When given red and yellow are additively combined, the result is the same orange regardless of the fact that the yellow (or the red) be spectral or not, as long as it looks the same; that is, mixtures of metameric colours are metameric. The tristimulus values of an additive mixture are the sums of the tristimulus values of the components $^\dagger$. This is a consequence of the psychophysical fact that our visual system, and in particular the photoreceptors in our retinae, work linearly within a moderate range of intensities.

Arguably, colour spaces geometrically represent all the colours we can perceive; we also make here the hypothesis that when two light beams are combined you can predict the colour of the resulting mixture on the mere basis of the colours being added.

The cone receptors L, M and S map each light spectrum into a set of three (spectral) responses; these responses can be quantified on the basis of the spectral absorbance of the corresponding photopigments. It is a strong reduction of information, to go from a spectral curve to a triple of numbers, yet it is informatoin we get usufruct from in highly advantageous ways. The sensorial mapping from the physical world to the world of quale is of the type many-to-one. Because the receptoral response is the only input to the visual system, things undistinguishable at the receptoral level are visually indistinguishable. Colour spaces are tridimensional because of the so-resultant trichromacy of our visual system.

$^\dagger$The components of RGB color space implicitly encode luminance (e.g. as max(R,G,B)) and represent irradiance; in this sense, because of the boundedness of the RGB cube, the RGB components are unlike CIE primaries, which obey ordinary addition and can grow indefinitely.
We take the standpoint that the RGB values of a pixel code the colour that you see when you watch an RGB image and that they represent the physical properties of the corresponding light ray in the scene, such as irradiance, spectrality and spectral contents. It is with those attributes that we see that we work with here.

Double-cone colour space has the advantage of being intuitive making geometrically explicit the attributes of luminance, hue and chromatic saturation and also of not normalizing the saturation by the luminance, forcing a decrease of the saturation at large and small values of the luminance. Only the equator of the double cone has colours fully saturated.

2. ON LUMINANCE, SATURATION AND HUE

RGB code is used electronics imaging, in cameras as well as in projectors and screens. Being based on the trichromacy of our visual system, RGB code is related to the receptor properties of the retina, when projecting an image. Also, RGB code is related to the physics of the light, as the pixels in the camera sensor transduce light into an electrical response. RGB space is readily translated into Double-Cone space (see Section 3.4), making explicit the intuitive properties of hue, saturation and luminance. The qualia of luminance relates to the physical property of the irradiance, or power per area.

We take the stand that the luminance is related to the irradiance via a saturation function such as the Naka-Rushton function; that the hue is circularly averaged (as opposed to linearly averaged), and that the chromatic saturation decreases for two reasons when two light beams are combined: the larger the difference between the hues being combined, the more the resulting mixture is desaturated, and also, the saturation decreases when achromatic light is added.

2.1 Luminance and luminance combination

A luminance function can be derived from the RGB values in several ways. A most readable form results with max(R,G,B) (which is the value of Matlab’s function hsv); interestingly, the min is also pretty readable. The map \((R,G,B) \rightarrow (R - \min(R,G,B), G - \min(R,G,B), B - \min(R,G,B))\) provides a "projection" onto the dark corner of the RGB cube, by moving each colour in a direction parallel to the achromatic segment, until it touches the dark corner; the resulting image is interesting to watch since \((\min(R,G,B), \min(R,G,B), \min(R,G,B))\) gives a measure of achromatic contents. The range is a measure of unnormalized (by the luminance) chromatic saturation and is also very readable. The geometric mean is pretty good too, and of course is related to the sum of the logarithms. See Fig. 2; the fact that one of these luminance images may look bad does not necessarily mean it be useless for other purposes, or that it does not carry important information. Here, we use the midrange as a measure of luminance, and the range as a measure of chromatic saturation.

2.1.1 Naka-Rushton Law

We assume a relation between the luminance and the corresponding irradiance of a light beam of the sigmoidal, or saturation, type, such as a Naka-Rushton curve; see Fig. 3. This is used to get the luminance of the combined beam by adding the irradiances, and to appropriately weigh the hue and chromatic saturation of the combining beams.

Naka-Rushton equation is a model for the intensity response \(V\) of photoreceptors: 
\[
\frac{V}{V_{\max}} = \frac{I^m}{I + K}, \quad \text{where } I \text{ is the stimulus radiance (photons/(cm}^2\text{s)}) V \text{ is the response amplitude at irradiance } I, V_{\max} \text{ is the maximum response amplitude, } m \text{ is the slope of the linear part of the } V/\log I \text{ curve and } K \text{ is the stimulus irradiance evolving a response that was } 50\% \text{ of the } V_{\max}. \text{ We work with the simplified function } f : [0, \infty) \rightarrow [0, 1] \text{ given by } y = f(x) = \frac{x}{x + 1}, \text{ with inverse } x = f^{-1}(y) = \frac{y}{1 - y}, \text{ } y \in [0, 1]; \text{ see Fig. 3.}
\]

We define an addition of sorts for luminances, that corresponds to addition of irradiances: 
\[
y_1 \oplus y_2 := f(f^{-1}(y_1) + f^{-1}(y_2)) = \frac{y_1 + y_2 - 2y_1y_2}{1 - y_1y_2}; \text{ note that } y \oplus y = \frac{2y}{1 + y^2}. \text{ In the appendix, we consider the more elaborated version of this addition. Since, to } y = 1 \text{ there corresponds an irradiance } x = \infty, \text{ in the Matlab programs we use, we subtract the quantity } 0.000001 \text{ from each of the components } R, G \text{ and } B, \text{ when they are nonzero.}
\]

\footnote{This of course would lead us to taking into consideration simultaneous induced contrast as well, which we do not at this stage.}

\footnote{Images are not supposed to be overexposed or under exposed.}
2.1.2 Associativity of conjugation

Since the operation of addition of luminances is an algebraic group conjugation, we have its associativity; in fact,

\[(a \oplus b) \oplus c = f(f^{-1}(a) + f^{-1}(b)) + f^{-1}(c) = (f^{-1}(a) + f^{-1}(b)) + f^{-1}(c) \]

\[= a \oplus b \oplus c \text{ and, in the same way, } a \oplus (b \oplus c) = : a \oplus b \oplus c.

Even though there may be effects of adaptation that make the result dependant on the order in which light beams are combined, a sort of hysteresis, we take the standpoint here that combination of light beams obeys associativity.

2.2 Spectrality and saturation

It is commonly assumed that the most saturated colours are the "spectral colours" (i.e. those of lights of a single wavelength). In the RGB cube, the chromatic hexagon,\(^2\) which corresponds to range \(\rho = 1\), is the set of points of maximal saturation; the chromatic hexagon corresponds to the equator of the double cone. Nevertheless, for the extraspectral hues, there are also fully saturated versions: those in the straight line segment connecting the two endpoints in CIE’s horseshoe diagram. Also, it is usually said that saturation is lost by the addition of a white light.

We perceive white or achromatic so long as the three receptors L, M and S are equally stimulated. The narrower a spectrum is, the lesser the probability that the three cones are stimulated. If only two of the cone types are stimulated, full saturation is possible, thus, if the L and M cones are stimulated (but not S) you see oranges and yellows, if the L and S but not M cone types are stimulated you see red and purples\(^\dagger\), and if the M and S but not L cone types are stimulated you see green, blues and cyans.

The use of the midrange and range as measures of luminance and saturation, respectively, implies that for each luminance value, the maximal saturation increases linearly from zero at zero luminance, to one at half maximal luminance, and then decreases back to zero at full luminance.

2.3 Hue: circular averaging and desaturation

Hue is a circular variable and it a circular weighted average, weighted by irradiance, is a good model for hue combination. On the other hand, the closer to \(\pi\) the difference between the angle hues is (\(\pi\) is the maximal possible difference), the more desaturated the result is.

3. COLOUR COMBINATION

It is useful to have a means to predict the colour that will result when two beams of light converge on a white surface, for prediction and simulation purposes, e.g. in design or in virtual reality. Given the RGB coordinates of the colours of the light beams, we propose an arithmetic model to get the colour coordinates of the combination. We work in double-cone space, which is readily derived from RGB cubic space. RGB itself is perhaps no an appropriate space to define the combination of two colours; you cannot use the parallelogram law, and (linear) averages are not always a good idea. Consider Figure 8.

3.1 Colour spaces and colour combination

We take the standpoint that, e.g. regarding luminance, the RGB values of a pixel represent the color that you see when you watch an RGB image. It is with those represented colors that we want to work with. Double-cone space is a colour space of the type hue-saturation-luminance. The proposed model gives the hue, the saturation and the luminance of the combination, provided those of the rays that are being combined.

3.2 CIE and RGB

RGB is a bounded space; CIE is not, in the sense that you can indefinitely add tristimuli. It is because \(x y\) CIE chromatic plane is obtained by projecting the primaries that a bounded space results. CIE \(x y\) (bounded) chromatic plane is obtained as \((X Y Z) \mapsto (x y z) = \frac{1}{X + Y + Z} [X Y Z].\)

\(^\dagger\)At the retinal/neuronal level, it does not matter that the L and S cells have disjoint bandwidths and there is no impediment to combine them. This in fact provides the closure of the hue circle. And yet, what survival advantage does a hue circle report?
3.2.1 On the projection $[X Y Z] \mapsto [x y z] = \frac{1}{X+Y+Z}[X Y Z]$

Consider first the analogous case in dimension two; i.e. consider the transformation $[X, Y] \mapsto [x, y] = \frac{1}{X+Y}[X, Y]$. You radially project onto the line $x + y = 1$, with projecting point given by the origin $(0, 0)$. Notice that each point of a line $[X, mX]$ ends up at the point $\left[\frac{1}{1+m}, \frac{m}{1+m}\right]$ of the line $x + y = 1$. The point $(0, 0)$, and in fact each point on the line $X + Y = 0$, has no finite image; the $Y$ axis ends up at the point $(0, 1)$ of the line $x + y = 1$. Also notice that each line $[X, mX]$ (or ray, if we consider the first quadrant only) intersects the line $Y = 1$ at $\left[\frac{1}{m}, 1\right]$ and the line $X = 1$ at $[1, m]$; both of these points are in turn mapped to the point $\left[\frac{m}{1+m}, \frac{m}{1+m}\right]$ of the line $x + y = 1$. Now, for the line $[X, mX]$, consider the intersection on the line $X = 1$ when $m < 1$, and the intersection the line $Y = 1$ when $m > 1$. In this sense, you radially (and injectively) project, first, onto the square angle formed by the lines $X = 1$ and $Y = 1$, and then, project the points on this angle onto the line $x + y = 1$; a sort of flattening of the angle into a staright line. Note that $[1, 1] \mapsto [1, 1]$. See Fig. 5.

Similarly, for the mapping $[X Y Z] \mapsto [x y z] = \frac{1}{X+Y+Z}[X Y Z]$, each line $[at, bt, ct]$ gets mapped to the point $\frac{1}{a+b+c}[a, b, c]$ in the plane $x + y + z = 1$. The point $[a/c, b/c, 1]$ in the plane $Z = 1$, as well as the points $[1, b/a, c/a]$ and $[a/b, 1, c/b]$ in the planes $X = 1$ and $Y = 1$, get mapped to the point $\frac{1}{1+b+c}[a, b, c]$ of the plane $x + y + z = 1$. Thus, you project first onto the corner formed by the planes $X = 1$, $Y = 1$ and $Z = 1$, $0 \leq X, Y, Z \leq 1$ and then flatten the corner on the plane $x + y + z = 1$. The CIE "spectral" primaries $X(\lambda), Y(\lambda)$ and $Z(\lambda)$ produce the line $[X(\lambda), Y(\lambda), Z(\lambda)]$ which in turn projects to the CIE $xy$ chromatic plane. See Fig. 6.

3.3 Double-cone space

The range $\rho$ of the RGB triple, being a measure of the distance to the achromatic segment in the cube, measures chromatic saturation while the midrange $\mu$, a measure of the distance to the black point in the cube, measures the luminance. The saturation in double-cone space is zero both for zero luminance and for maximal luminance. We work with double cone space which has a saturation that is not luminance-normalized, as many times occurs. It forbids large amounts of saturation for large and small values of the luminance. Double-cone space is a solid of revolution; the revolution surface is a triangle, the $\mu\rho\mu$ triangle, about its base (the $\mu$ axis).

3.3.1 The surfaces $\mu = constant$ and $\rho = constant$

In the first octant of $\mathbb{R}^3$, $\rho = constant$ is a "tube" (or boundary of the infinite-length prism) with core given by the achromatic line, and of hexagonal perimeter; while the solid $\mu \leq constant$ is caped off by a 6-triangle, non-convex surface of positive curvature. The non-convex surface $\{ (x, y, z) \in \mathbb{R}^3 | midrange(|x|, |y|, |z|) = k \}$ is an isohedron of $8 \times 6$ triangular faces, the triangles being isosceles and of two size types, known as the first stellation of the rhombic dodecahedron. See Fig. 7.

3.4 Routines erm2rgb and rgb2erm

Matlab routines for converting between RGB and double cone coordinates $\eta\rho\mu$ are shown below.

```matlab
function [B] = rgb2erm(A)
    [ORDERED, RANK] = sort(A);
    B(2) = (ORDERED(3)+ORDERED(1))/2;
    B(3) = ORDERED(3)-ORDERED(1);
    if B(3)==0
        B(1)=1; %undefined hue
    else
        ORDERED = (ORDERED-ORDERED(1))/B(3); %project onto chromatic hexagon
        if RANK(1)==2 && RANK(2)==2; B(1)=0; end
        if RANK(1)==3 && RANK(2)==1; B(1)=1; end
        if RANK(1)==1 && RANK(2)==3; B(1)=2; end
        if RANK(1)==2 && RANK(2)==1; B(1)=3; end
        if RANK(1)==3 && RANK(2)==2; B(1)=4; end
        if RANK(1)==1 && RANK(2)==2; B(1)=5; end
        if A(1)==1 && A(2)==0 && A(3)==0; B(1)=0; end
        if A(1)==1 && A(2)==1 && A(3)==0; B(1)=1; end
        %ranks may be ambiguous at vertices
    end
end
```
if \ A(1)==0 \&\& \ A(2)==1 \&\& \ A(3)==0; \ B(1)=2; \end
if \ A(1)==0 \&\& \ A(2)==1 \&\& \ A(3)==1; \ B(1)=3; \end
if \ A(1)==0 \&\& \ A(2)==0 \&\& \ A(3)==1; \ B(1)=4; \end
if \ A(1)==1 \&\& \ A(2)==0 \&\& \ A(3)==1; \ B(1)=5; \end
if \ B(1)==0 \|\| \ B(1)==2 \|\| \ B(1)==4
B(1)= \ B(1)+ \ \text{ORDERED}(2); \ \% \ \text{hexagon\_side.median}
else
B(1)= B(1)+(1-\text{ORDERED}(2)); \ \% \ \text{hexagon\_side.median}
end
end;

function[B] = erm2rgb(A)
if A(1) = -1
if floor(A(1))==0 \ % \ \text{RGB}= 1X0
H=[1, \ \text{mod}(A(1),1),0];
end
if floor(A(1))==1 \ % \ \text{RGB}= X10
H=[1-\text{mod}(A(1),1), 1, 0];
end
if floor(A(1))==2 \ % \ \text{RGB} = 01X
H=[0,1, \ \text{mod}(A(1),1)];
end
if floor(A(1))==3 \ % \ \text{RGB} = 0X1
H=[0, 1-\text{mod}(A(1),1), 1];
end
if floor(A(1))==4 \ % \ \text{RGB} = X01
H=[\text{mod}(A(1),1), 0, 1];
end
if floor(A(1))==5 \ % \ \text{RGB} = 10X
H=[1, 0, 1-\text{mod}(A(1),1)];
end
BETA= A(2)-0.5*A(3); \ % \mu-0.5\rho
GAMMA= A(3); \ % \rho
B= BETA*[1,1,1] + GAMMA*H;
else
B= [A(2), A(2), A(2)];
end

4. A MODEL FOR COLOUR COMBINATION

Suppose that colours \((\eta_1, \mu_1, \rho_1)\) and \((\eta_2, \mu_2, \rho_2)\) are to be combined.

PROCEDURE:

1. Both colors are achromatic: \(\eta_1 = * \) (undefined), \(\rho_1 = 0\) and \(\eta_2 = * \) (undefined), \(\rho_2 = 0\).
   
   The resulting colour blend \((*, \mu, 0) = (*, \mu_1, 0) \oplus (*, \mu_2, 0)\) is achromatic and the luminance \(\mu\) is given by \(f(f^{-1}(\mu_1) + f^{-1}(\mu_2))\).

2. One colour \((*, \mu_1, 0)\) is achromatic while the other one \((\eta_2, \mu_2, \rho_2)\) is chromatic.

   The resulting colour \((\eta_2, \mu, \rho) = (*, \mu_1, 0) \oplus (\eta_2, \mu_2, \rho_2)\) has the same hue as the chromatic colour, has luminance \(\mu\) given by \(f(f^{-1}(\mu_1) + f^{-1}(\mu_2))\) and has saturation \(\rho\), decreased from \(\rho_2\) by the factor \(\frac{f^{-1}(\mu_1) + f^{-1}(\mu_2)}{f^{-1}(\mu_2)}\).
3. Both colours \((\eta_1, \mu_1, \rho_1)\) and \((\eta_2, \mu_2, \rho_2)\) are dark-version\(^{\text{II}}\), fully chromatic: \(\mu_1 = \rho_1/2\) and \(\mu_2 = \rho_2/2\).

Combine luminances: \(\mu = f(f^{-1}(\mu_1) + (f^{-1}(\mu_2)))\). Then, to a corresponding full saturation of \(\rho_F = 1 - |2\mu - 1|\), apply a desaturation factor \(K := \left(\frac{1}{2}||\eta_1 - \eta_2| - \pi||\right)\gamma\) that takes into account how different the hues are (\(\gamma \in [0, 1]\) and \(\eta_1, \eta_2 \in [0, 2\pi]\)); the maximal difference is of course \(\pi\). Thus, \(\rho = K\rho_F\). Finally, the hue \(\eta\) of the combination is obtained as a circular weighted average with weights given by the irradiances \(\eta = \frac{\rho F - 1}{2}\). The maximal difference is \(\pi\). Thus, \(\rho = K\rho_F\).

4. Both colours \((\eta_1, \mu_1, \rho_1)\) and \((\eta_2, \mu_2, \rho_2)\) are chromatic.

To compute their colour blend \((\eta, \mu, \rho)\) where \(\mu = f(f^{-1}(\mu_1) + (f^{-1}(\mu_2)))\), decompose each colour into a fully saturated, chromatic component and an achromatic component. Thus, \((\eta_1, \mu_1, \rho_1) = (\eta_1, \mu_{11}, \rho_{11}) \oplus (*, \mu_{12}, 0)\), where \(\rho_{11} = \rho_1\) (same saturation), \(\mu_{11} = \rho_1/2\) (darker option of fully saturated versions of same saturation), and \(\mu_{12} = f(f^{-1}(\mu_1) - f^{-1}(\rho_1/2))\). Analogously for \((\eta_2, \mu_2, \rho_2) = (\eta_2, \mu_{21}, \rho_{21}) \oplus (*, \mu_{22}, 0)\).

Then, combine the two achromatic colours \((*, \mu_{12}, 0) \oplus (*, \mu_{22}, 0)\), as in 1, and the two fully chromatic chromatic colours \((\eta_1, \mu_{11}, \rho_{11}) \oplus (\eta_2, \mu_{21}, \rho_{21})\), as in 3. Finally, combine the chromatic and the achromatic resultants as in 2.

Below, the corresponding Matlab code is given.

```matlab
function[C] = BlendDblcne(A,B)
D=zeros(150,100,3);
ifAA= rgb2erm(A)
BB= rgb2erm(B)
COP=AA(3); %order messed
AA(3)= AA(2);
AA(2)= COP;
COP=BB(3);
BB(3)= BB(2);
BB(2)= COP;
if AA(3) = 0
AA(3)= AA(3) - 0.0000001;
end% NakaInv is infinity for 1
if BB(3) = 0
BB(3)= BB(3) - 0.0000001;
end% NakaInv is infinity for 1
C(3)= Naka(NakaInv(AA(3))+NakaInv(BB(3)));
if AA(1)==-1 && BB(1)==-1 % if both colours are achromatic
C(1)= -1;
C(2)= 0;
else
if AA(1)==-1 % if only A is achromatic (B can’t be black so BB(3)?0)
C(1)= BB(1); % same hue
C(2)= BB(2)*NakaInv(BB(3))/(NakaInv(BB(3))+NakaInv(AA(3))); %dilute saturation
else if BB(1)==-1 % if only B is achromatic (A Can’t be black)
C(1)= AA(1); % same hue
C(2)= AA(2)*NakaInv(AA(3))/(NakaInv(BB(3))+NakaInv(AA(3))); %dilute saturation
else % if both A and B are chromatic
AAchr(1)= AA(1); % decompose into achromatic pluss dark fully chromatic
BBchr(1)= BB(1);
```

\(^{\text{II}}\)Unless \(\rho = 1\), which uniquely corresponds to \(\mu = 0.5\), to each maximal value of \(\rho\), there correspond two values of \(\mu\): \(\rho/2\) and \(1 - \rho/2\). The "darker version" is that with \(\mu = \rho/2\).
AAchr(2) = AA(2);
BBchr(2) = BB(2);
AAchr(3) = AA(2)/2;
BBchr(3) = BB(2)/2;
Cchr(3) = Naka(NakaInv(AAchr(3)) + NakaInv(BBchr(3)));
CchrTemp = 1 - abs(2*Cchr(3)-1); % corresponding full saturation
DE2 = 1 - abs(2*abs(AA(1)/6 - BB(1)/6)-1); % twice the shortest difference between angles mod-6
gamma = 1; % can adjust desaturation factor in several ways besides gamma correction
Cchr(2) = CchrTemp * (1-DE2)^gamma;
Factor = NakaInv(AAchr(3)) + NakaInv(BBchr(3));
Theta1 = 1i*2*pi*AA(1)/6;
Theta2 = 1i*2*pi*BB(1)/6;
Complex1 = exp(Theta1);
Complex2 = exp(Theta2);
FF1 = NakaInv(AAchr(3));
FF2 = NakaInv(BBchr(3));
Complex = (FF1*Complex1 + FF2*Complex2)/Factor;
Angle = mod(angle(Complex),2*pi);
Normangle = Angle/(2*pi)*6;
Cchr(1) = Normangle; % in ERM space angles are mod-6
if DE2 == 1
    Cchr(1) = -1;
end
AAachr(3) = AA(3) - AA(2)/2; % mu - rho/2 is achromatic component
BBachr(3) = Naka(NakaInv(BB(3)) - NakaInv(BB(2)/2));
Cachr(3) = Naka(NakaInv(AAachr(3)) + NakaInv(BBachr(3)));
Cachr(1) = -1;
Cachr(2) = 0;
C(1) = Cchr(1); % same hue
C(2) = Cchr(2)*NakaInv(Cchr(3))/(NakaInv(Cchr(3)) + NakaInv(Cachr(3))); % dilute saturation
end
end
CC(1) = C(1);
CC(2) = C(3); % order messed up
CC(3) = C(2);
CC = erm2rgb(CC);
for ii = 1:50
    for jj = 1:100
        D(ii,jj,1) = A(1);
        D(ii,jj,2) = A(2);
        D(ii,jj,3) = A(3);
    end
end
for ii = 51:100
    for jj = 1:100
        D(ii,jj,1) = CC(1);
        D(ii,jj,2) = CC(2);
        D(ii,jj,3) = CC(3);
    end
end
for ii = 101:150
5. Change of $\rho$ and $\mu$ in Double-Cone Space

For the modification of the saturation we have $\rho \mapsto \frac{1-|2\mu-1|}{1-|2\mu|} \rho^\gamma$, so that $(\mu, \rho) \mapsto (\mu, \frac{1-|2\mu-1|}{1-|2\mu|} \rho^\gamma)$, a sort of exponential-law ("gamma") correction that respects the boundary of the triangle. For luminance correction we also have an sort of exponential-law correction but we also change somewhat the saturation in order to stay within the triangle; we have $(\mu, \rho) \mapsto (\mu^\gamma, \frac{1-|2\mu-1|}{1-|2\mu|} \rho)$. The lines of unnormalized saturation, shown in Fig. 10 are followed.

6. Conclusion

Arguably, there are three types of colour mixtures: additive colour mixture, that results when lights are combined, subtractive colour mixture or combination of pigments (considered by Aristotle in the case when white light pases through two colored glasses), and also the mixture of the colour of a light with the colour of a surface that reflects it, before reaching the eye.

A surface that reflects light behaves, in a sense, like a transparent medium that filters light. Let $S \in [0,1]$ stand for spectrum, $R \in [0,1]$ stand for reflectance and $A = 1 - R$ stand for absorbance. In an additive mixture, $S_M = S_1 + S_2$; in a subtractive mixture, $R_M = R_1 R_2$ or, equivalently, $A_M = A_1 + A_2 - A_1A_2$. In the mixture of the colour of a light and the colour of a surface, you have $S = S_1 R_2$.

Additive mixtures, involving the colours of two lights, are less intuitive because, in practice, there is less opportunity of changing the colour of a light at will. We have presented a model for the combination of light colours using double-cone space, as well as the corresponding tool for the prediction of the colour of an additive colour mixture.

References


Appendix

With, $I = K \left( \frac{V}{V_{max}} \right)^{1/m} = K \left( \frac{V}{V_{max}/V-1} \right)^{1/m}$

Let $I' = I_1 + I_2 = K \left( \frac{V}{V_{max}-V_1} \right)^{1/m} + K \left( \frac{V}{V_{max}-V_2} \right)^{1/m}$

with corresponding $V' = V_{max} \left( \frac{V}{V_{max}} \right)^{1/m} + K m$

$= V_{max} \left( \frac{V}{V_{max}-V_1} \right)^{1/m} + K \left( \frac{V}{V_{max}-V_2} \right)^{1/m}$

$= V_{max} \left( \frac{V}{V_{max}-V_1} \right)^{1/m} + K \left( \frac{V}{V_{max}-V_2} \right)^{1/m}$

with $V_{max} = 1$,

$= \left( \frac{V}{V_{max}-V_1} \right)^{1/m} + \left( \frac{V}{V_{max}-V_2} \right)^{1/m}$

$= \left( \frac{V}{V_{max}-V_1} \right)^{1/m} + \left( \frac{V}{V_{max}-V_2} \right)^{1/m}$

$= \left( \frac{1}{V_{max}-V_1} \right)^{1/m} + \left( \frac{1}{V_{max}-V_2} \right)^{1/m}$

$= \left( \frac{1}{V_1} \right)^{1/m} + \left( \frac{1}{V_2} \right)^{1/m}$. 
Figure 1. Luminance measurements of Image Venezia; the min, the midrange, the max, the arithmetic average, the Euclidean distance from black, and the geometric average.
Figure 2. Saturation measurement given by range. At the right side, the result of subtracting from each pixel in the image, \((R, G, B) - (\text{min}, \text{min}, \text{min})\).

Figure 3. The Naka-Rushton saturation curve \(f : [0, \infty) \rightarrow [0, 1), f(x) = \frac{x}{1+x}\).

Figure 4. The colours at the top and the bottom of each rectangle are mixed in the intermediate patch. In the three rectangles at the left, the mixture is implemented with separate averages of R, G and B components of the colours. In the two rectangles at the right, circular average of the hues, average of the saturation (range) and "radiant composition" of luminance (midrange) is used.
Figure 5. The projection of a point in two steps: first project on the square angle, then on the diagonal line.

Figure 6. The image $[X(\lambda), Y(\lambda), Y'(\lambda)]$ of the CIE primaries $XYZ$, and projection, $xy$ horseshoe diagram.
Figure 7. Nonconvex boundary of constant luminance (midrange) sets in RGB space (midrange($|R|$, $|G|$, $|B|$), in the first octant.)

Figure 8. Each pair of consecutive rectangles shows average RGB combination and colour combination according to the method introduced here.
Figure 9. Each pair of consecutive rectangles shows average RGB combination and colour combination according to the method introduced here.
Figure 10. Each pair of consecutive rectangles shows average RGB combination and colour combination according to the method introduced here.